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A Method of Search for Leaky Waves Based on Exceptional Wave Theory

N.F.Naumenko

Moscow Steel & Alloys Institute, 117936, Leninski prosp.,4. Moscow, Russia

Abstract - A method of search for leaky waves in crystals is based on the calculation of permitted orientations for exceptional bulk waves, which satisfy the stress-free boundary conditions for surface waves. This method is applied to quartz. The problem of existence is discussed for quasi-shear and quasi-longitudinal exceptional waves in three cut families of trigonal crystals: (0°,θ,0°), (90°,90°,ψ) and (0°,θ,90°).

I. INTRODUCTION

Leaky or pseudo-surface wave is a solution of surface acoustic wave (SAW) problem with complex velocity value resulting in attenuation of wave amplitude along the propagation direction. Leaky waves travel faster, than usual Rayleigh type SAW, since their velocity $V>V_c$, where V_c is the cut-off velocity for bulk waves in the specified orientation. In some crystallographic cuts the attenuation of leaky waves can be negligible providing possibility of their application in SAW devices. Due to higher propagation velocities, better temperature behavior and potentially higher piezoelectric coupling while implemented in SAW devices leaky waves can provide better device parameters, than Rayleigh-type SAW.

On the contrary to Rayleigh SAW, leaky waves exist only in permitted orientations. The search for these orientations by means of numerical methods, used for SAW investigation in the specified crystal cuts (for example, Farnell’s technique [1]), is very long and laborious work. However, leaky waves with quasi-bulk structure, similar to existing in 41°-YX cut of $LiNbO_3$ and 37°-YX cut of $LiTaO_3$ can be easily found using exceptional wave (EW) analysis.

Theory of EW, being the part of SAW theory, was considerably developed during the last two decades (see, for example [2-6]). Though the main results were obtained for pure elastic media they can be successfully applied to piezoelectric crystals.

II.METHOD OF EXCEPTIONAL WAVE LINES

In arbitrary crystallographic cut, in general, none of three surface skimming bulk waves (SSBW) propagating along the surface of pure elastic half-space satisfies the stress-free boundary conditions. However, it is known, that if the bulk wave propagates in the plane of material symmetry and has polarization orthogonal to this plane, such bulk wave can be SAW solution, called shear horizontally polarized (SH) wave. V.I.Alshits and J.Lothe [3] have shown, that a bulk wave with homogeneous polarization **u**, traveling in non-symmetric direction **n**, also can leave the selected boundary surface stress-free, if

$$\det(\mu_{ij})=0, \qquad (1)$$

where

$$\mu_{ij} = c_{ijkl}\, u_k\, n_l \qquad (2)$$

and c_{ijkl} is the elastic stiffness tensor.

Such bulk waves were called exceptional [2]. If one scans the three-dimensional space of propagation directions **n** for bulk waves, one-dimensional sub-space can be found where one of three bulk waves is exceptional. Each EW has the selected boundary plane which is normal to vector **m** defined by equation [3]

$$\mu_{ij}\, m_j=0. \qquad (3)$$

On the stereographic projection of unit wave vectors sphere $\mathbf{n}^2=1$ the permitted directions for EW may be plotted as the exceptional wave lines (EWL). Their configuration depends on the symmetry of crystal and the position of acoustic axes[3,4]. The numerical calculations of EWLs in some crystals of practical interest [4,7,8] have shown that practically in any crystal such lines can be found for slow and fast quasi-shear bulk modes, while quasi-longitudinal EWs exist only in crystals with strong elastic anisotropy, for example, paratellurite TeO_2, lithium tetraborate $Li_2B_4O_7$ and quartz.

EW is one-partial homogeneous solution of SAW problem. The unit wave vector of EW **n** is usually tilted with respect to the boundary plane at an angle ξ, but the power flow and polarization vectors always lie in the boundary plane[3]. Dependent on the polarization angle β between the propagation direction of SAW $\mathbf{n}' = \mathbf{m}\times(\mathbf{n}\times\mathbf{m})$ and its polarization **u,** homogeneous SAW solution can be SH (β=90°), quasi-SH (45°<β<90°), quasi-LH (0°<β<45°) or LH (pure longitudinal, β=0°). Two last types of SAW are characterized by high propagation velocity, 1.5...2 times higher than that of Rayleigh SAW.

Piezoelectric effect can be taken into account while calculating EWL if the effective values of elastic stiffness constants are used in (1)-(2) and polarization vectors of bulk waves are found for piezoelectric medium. The electric boundary conditions usually slightly change the structure of SAW, making it quasi-bulk and quasi-horizontally polarized. If the velocity of wave exceeds V_c it becomes leaky. The parameters of modified EW can be estimated by means of known numerical techniques.

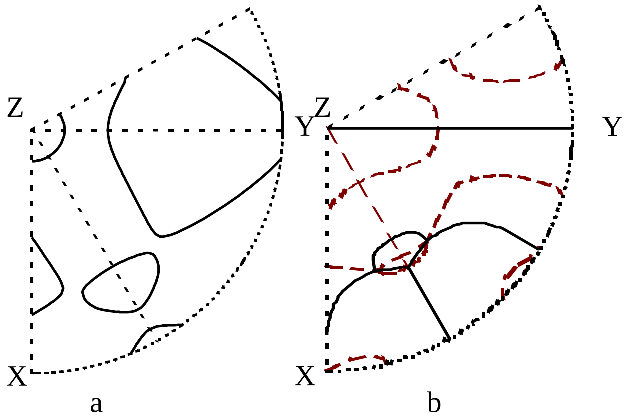
This method was first applied to paratellurite TeO_2 [8] and it was discovered that in some orientations in this crystal quasi-longitudinal bulk wave propagates without attenuation along the free surface of half-space. Later similar waves were found and investigated in $Li_2B_4O_7$ [4,9-11], where they are transformed to piezoelectrically coupled leaky waves with small or even zero attenuation. Recently the existence of quasi-longitudinal leaky waves in $Li_2B_4O_7$ was verified experimentally [12].

In the present work the method is applied to quartz .

III. APPLICATION OF EWL METHOD TO QUARTZ

Fig.1 shows the calculated EWLs on the stereographic projection of unit wave vector sphere for quartz. EW orientations have been discovered for all three bulk modes. For convenience, quasi-longitudinal EWLs (fig.1a) are plotted separately from fast and slow quasi-shear ones (fig.1b). The large number of lines is caused by the strong elastic anisotropy of crystal. In addition to acoustic axes (bold points), located in the planes of material symmetry and typical for trigonal crystals, quartz has some axes “of general position” and consistently with EW theory [3,4] in these points EWL must go from one degenerate branch to another. The term “planes of material symmetry” is referred in quartz (point symmetry 32) to the planes orthogonal to 2-fold symmetry axes and in $LiNbO_3$ and $LiTaO_3$ (point symmetry 3m) to the planes of reflexional symmetry.

Each point of EWL shows one propagation direction for exceptional waves. The corresponding normals to selected boundary planes also can be plotted as continuos lines. For quartz their configuration may be found in [4].



a b

Fig.1. Stereographic projection of exceptional wave lines in quartz: a - for quasi-longitudinal bulk mode(solid lines); b - for quasi-shear bulk modes (fast - dashed lines, slow - solid lines). Bold points show the acoustic axes.

For practical applications the most interesting orientations are symmetric ones or singly rotated cuts, for which at least two of three vectors - **n**, **m** and **u** - lie in the plane of material symmetry. In fig.1 symmetric orientations can be found as the points where EWL falls within the plane of material symmetry (dashed and solid straight lines in fig.1b) or goes through X axis. All symmetric orientations may be divided into three cut families with Euler angles (0°, θ, 0°), (90°, 90°, ψ) and (0°, θ, 90°). Let us consider the existence conditions and typical features of EW in each cut family.

A. Euler angles (0°, θ, 0°)

The propagation direction for these orientations is parallel to X axis. EW must be polarized in YZ plane. Two bulk shear waves, propagating along X axis and polarized in YZ plane both satisfy the criterion for EW (1). The selected boundary plane for each shear EW is parallel to its polarization **u** and X axis of crystal. Thus to find permitted orientations for EW among rotated Y-cuts, X-propagation, it is enough to calculate polarization vectors for two bulk shear waves, traveling along X. One of EW is always fast shear, the other is slow shear. Because of symmetry both SAW solutions are pure SH.

In quartz fast shear wave is exceptional for θ=121.6° and slow shear for θ=31.6°. Piezoelectric effect shifts these cuts only by 0.02°. In $LiNbO_3$ similar orientations are defined by θ=-49 and 41° and in $LiTaO_3$ by θ=-53 and 37°. In all three crystals fast shear EW gives rise to leaky wave branch. Fig.2 shows well known dependencies of SAW and leaky wave propagation velocities in (0°, θ, 0°) cuts of quartz. In two orientations where EWs exist, either SAW or leaky wave degenerates into

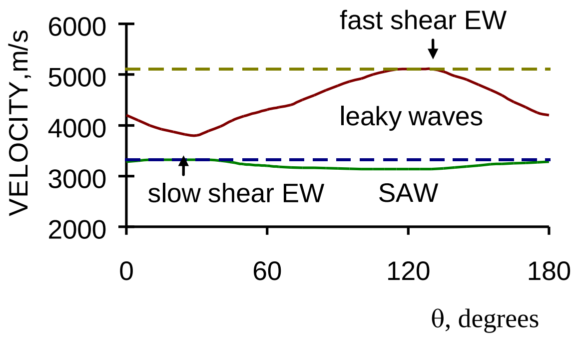


Fig.2. Propagation velocities of SAW, leaky and exceptional bulk waves in quartz cuts with Euler angles (0°,θ,0°).

SH-type (quasi)bulk wave. In $LiNbO_3$ both EWs are included in leaky wave branch. Probably this picture is typical for any crystal of symmetry class 32 or 3m.

B.Euler angles (90°, 90°, ψ)

EW propagates in YZ-plane and must be polarized in the same plane. Thus the solutions can be found among quasi-shear and quasi-longitudinal bulk waves. From equation (1) and Christoffel equation for bulk waves it can be derived, that in addition to acoustic axes there exist 2 or 4 directions, where EW can propagate.

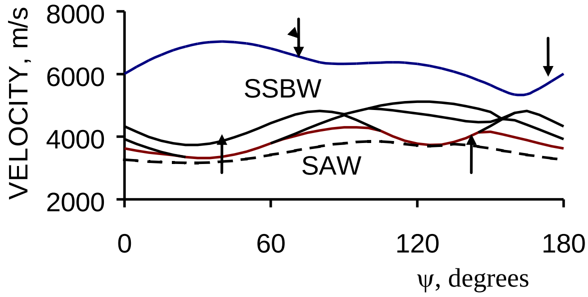


Fig.3. Propagation velocities of SAW (dashed line) and SSBW (solid lines) in quartz, Euler angles (90°, 90°, ψ). Exceptional waves are shown by arrows.

In quartz this number is the largest: quasi-shear waves are exceptional for ψ=41.2 and 145.6° and quasi-longitudinal - for ψ=75.1 and 176.7° (fig.3). In $LiNbO_3$ and $LiTaO_3$ only quasi-shear EWs have been found: ψ=23.1 and 69.8° for $LiNbO_3$ and ψ=28 and 112° for $LiTaO_3$ [7]. Leaky waves were proved to exist near one of these orientations in $LiNbO_3$[13].

C. Euler angles (0°, θ, 90°).

SH-waves are known to exist in this cut family for any θ. It is obvious, since for any bulk wave, propagating in YZ plane and polarized along X, det(μ)=0. Two other bulk modes, which are polarized in YZ plane, also can be exceptional if (1) is satisfied. In this case **n**, **m** and **u** belong to the same plane which means that β=ξ and the power flow vector of EW is codirectional with its polarization vector. Such bulk waves were called "ray-polarized" (RP) by P.Chadwick [5] who proved, that any RP wave is exceptional until it is not pure longitudinal.

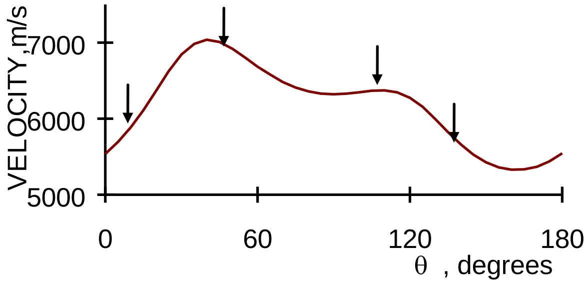


Fig.4. Propagation velocities of quasi-longitudinal SSBW in quartz cuts with Euler angles (0°, θ, 90°). Exceptional waves are shown by arrows.

Though both quasi-shear and quasi-longitudinal waves can be exceptional, the condition β=ξ in practice means that "ray-polarized" waves are quasi-longitudinal, because usually the tilt angle ξ<45°. So these surface waves have the highest propagation velocity.

Though EW in general is quasi-longitudinal, the corresponding SAW solution is pure longitudinal: its propagation direction lies in the boundary plane and hence is parallel to the polarization vector.

Pure longitudinal EW (β=ξ=0) can be exceptional only under special conditions. The analysis of these conditions [14] reveals a simple relation between the elastic moduli of trigonal crystal, permitting the existence of quasi-longitudinal EW with small polarization angles: $c_{44} >> c_{13}$. In quartz $r = c_{44} / c_{13} \approx 4.8$, in $Li_2B_4O_7$ $r \approx 1.8$. In both crystals quasi-longitudinal EWs exist. On the contrary, in $LiNbO_3$ ($r \approx 0.8$) and $LiTaO_3$ ($r \approx 1.17$) there are no such waves.

Four cuts where quasi-longitudinal bulk wave is exceptional are marked in fig.4: θ=15.8, 47.8, 105.8 and 137.8°. SAW is not piezoactive because of symmetry. However, piezoelectrically coupled leaky waves have been found near orientation (0°,137.8°,90°) (fig.5,6). On metallized surface attenuation coefficients δ of leaky waves in the area of their existence (fig.6) do not exceed 10^{-3} dB/wavelength. If the surface is free, these coefficients are even smaller and the permitted area becomes larger and includes ST,X+25° and even ST,X cuts with Euler angles (0°, 132.75°, 25°) and (0°, 132.75°, 0°) respectively (see, for example, [15]).

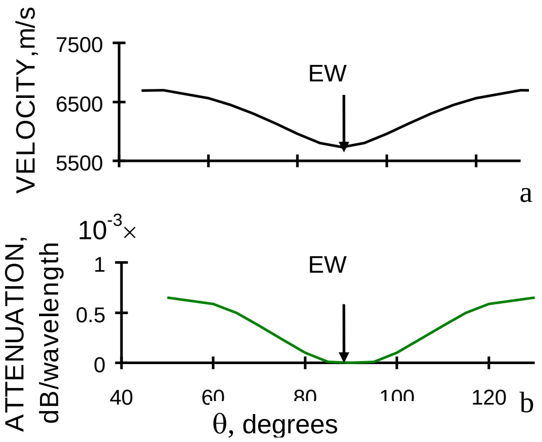


Fig.5. Propagation velocities (a) and attenuation coefficients (b) for quasi-LH leaky waves propagating on metallized surface of quartz, Euler angles (0°, 137.8°, ψ).

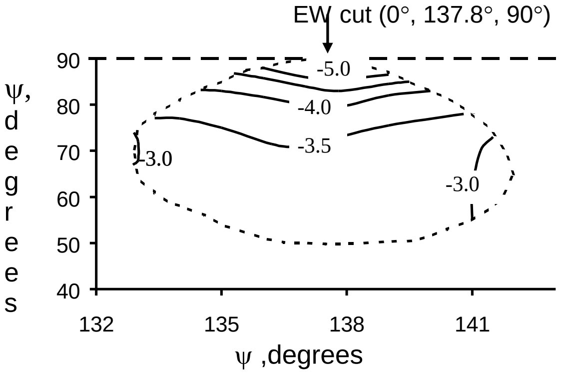


Fig.6. The area of existence and attenuation lg(δ) of quasi-LH leaky waves on metallized surface of quartz as function of two Euler angles in orientations (0, θ, ψ).

IV. CONCLUSIONS

The application of exceptional wave theory to piezoelectric crystals used in SAW devices gives a new insight of SAW and leaky waves and reveals a close relation between the symmetry and elastic anisotropy of the crystal and the existence of leaky waves in some orientations.

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