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The Behavior of Quasi-Longitudinal Leaky Surface Waves in Crystals

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Abstract - A numerical analysis of non-leaky surface waves propagating in crystals at the supersonic velocity has been performed. These waves were found to be composite bulk waves (CBWs), in general three-partial and comprising incident quasi-longitudinal and two reflected quasi-transverse bulk modes. A projection of the subspace, in which the CBW satisfies the stress-free boundary condition, was calculated for orientations of $LiNbO_3$, defined by the Euler angles (90°, 90°, ψ). The angle ψ=163.3°, for which CBW satisfies both mechanical and electrical boundary conditions, agrees with known propagation direction for non-leaky wave, and the angular dependence of velocity for CBW is very close to that of leaky waves, when the surface is non-metallized. Application of the same technique to quartz cuts (0°,θ,ψ) gave the lines of supersonic non-leaky waves. These lines include earlier found exceptional waves, which correspond to degeneration of the three-partial CBW into one-partial exceptional one.

I. INTRODUCTION

Quasi-longitudinal leaky (or pseudo-surface) waves, first proved to exist in $Li_2B_4O_7$[1,2], were recently found in many piezoelectric crystals [3,4]. This type of leaky waves is distinguished by the predominant contribution of quasi-longitudinal bulk mode and high propagation velocity, which results in carrying energy from the surface into the bulk by two partial modes instead of one, as in the case of common low-velocity leaky wave.

In order to perform a rapid search for low-attenuated leaky waves in any crystal, a special method was developed [5,6], based on the exceptional wave (EW) theory. All directions, for which one of three bulk modes satisfies the stress-free boundary condition, are calculated on account of crystal symmetry and peculiarities of its acoustic anisotropy. This method enabled to find non-leaky quasi-longitudinal waves in $Li_2B_4O_7$ and quartz [5-7]. Though electrical boundary conditions, while taken into account, lead to the transformation of one-partial EW into four-partial quasi-bulk solution, the attenuation coefficient of this modified EW is usually negligible for free (non-metallized) surface and the velocity is slightly lower than that of quasi-longitudinal surface skimming bulk wave (SSBW).

However, the exceptional wave approach cannot explain the existence of non-leaky surface waves in the velocity region $V>V_L$. These waves are called here “supersonic” on the contrary to “subsonic” SAW ($V< V_{T1}$) and leaky surface waves having propagation velocities in the first ($V_{T1}<V<V_{T2}$) and the second ($V_{T2}<V<V_L$) “transonic” regions [8], where V_{T1}, V_{T2} and V_L are respectively, the velocities of slow, fast quasi-transverse and quasi-longitudinal SSBWs. The numerical analysis of supersonic leaky waves, recently found in quartz [3] and $LiNbO_3$ [4], has been performed to explain their nature and to establish some general features of their behavior in non-piezoelectric and piezoelectric crystals.

II. ANALYSIS OF SUPERSONIC LEAKY WAVES IN QUARTZ

The application of EW method to quartz [5,6] gave six singly rotated cuts, where the quasi-longitudinal SSBW causes zero normal stresses while propagating along the surface, thus being EW. In the vicinity of orientation (0°, 137.8°, 90°) a crystallographic area was found[5], where piezoelectrically coupled leaky waves can propagate. Their velocities are slightly lower than V_L. On the boundary of this area $V= V_L$.

Since leaky waves have non-zero attenuation coefficients, when their velocities approach to V_L , one can expect, that these waves exist, when $V>V_L$. Indeed, supersonic leaky waves have been found in the cuts (0, θ,ψ) for all values of θ and ψ, with exception of mentioned above crystallographic area. Velocities V and attenuation coefficients δ for three different values of θ and ψ continuous are plotted in fig.1a,b. All leaky waves are supersonic, and for each θ the corresponding angle ψ exists, which gives leaky wave with negligible attenuation ($\delta<10^{-5}$ dB/λ). For example, in ST-cut (θ=132.75) non-leaky supersonic wave propagates, when ψ =29° [3].

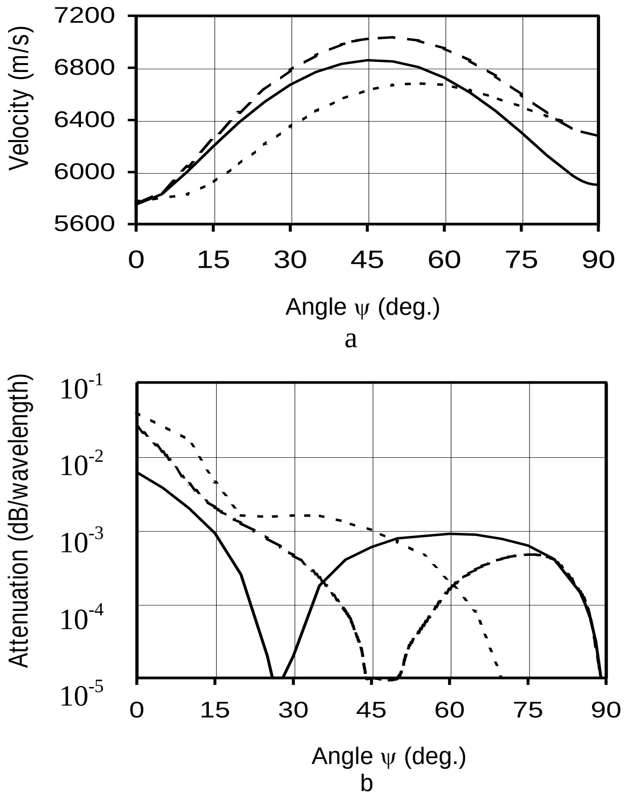

Fig.1. V*elocities (a) and attenuation coefficients (b) of leaky waves, propagating in quartz cuts (0, θ, ψ) versus angle ψ: θ=137.8° (solid lines), θ=120° (dashed lines), θ=100° (dotted lines).*

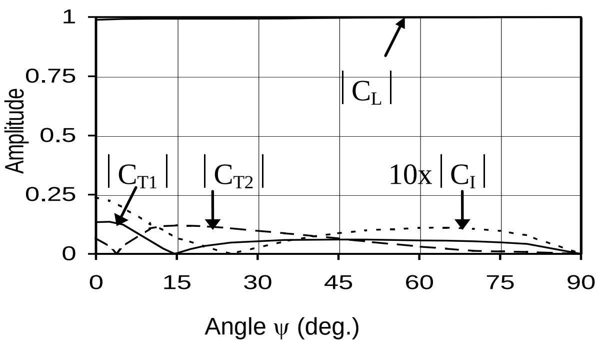

Fig.2. Relative amplitudes of partial modes, which compose supersonic leaky wave, versus propagation angle ψ in quartz cuts (0°, 137.8°, ψ°). The subscripts L, T1 and T2 refer to quasi-longitudinal and two quasi-transverse bulk modes, and the subscript I refers to inhomogeneous mode.

In general the supersonic SAW solution comprises three bulk (homogeneous) partial waves

and only one inhomogeneous wave:

$$\mathbf{U}=C_L\,\mathbf{U}_L + C_{T1}\,\mathbf{U}_{T1} + C_{T2}\,\mathbf{U}_{T2} + C_I\,\mathbf{U}_I\,, \quad (1)$$

where the subscripts L, T1, T2 refer respectively to quasi-longitudinal, fast and slow quasi-transverse bulk partial waves and the subscript I is associated with non-homogeneous wave. For supersonic leaky surface waves with small attenuation coefficients three bulk modes become quasi-bulk.

To clarify the nature of supersonic non-leaky waves, the contribution of each partial mode into the leaky solution was numerically analyzed in quartz cuts (0°, 137.8°, ψ°) (fig.2). The quasi-longitudinal bulk mode is predominant and for ψ=90° (EW direction) $|C_L|=1$, while the contribution of inhomogeneous mode is very small and in the point ψ =28° it vanishes, $|C_I|=0$. It means, that the supersonic non-leaky wave is composed of three bulk modes (L+T1+T2) and may be considered as the composite bulk wave (CBW). In the cut (0°, 120°, 50°) and other orientations, where δ=0, the amplitude coefficients exhibit similar behavior.

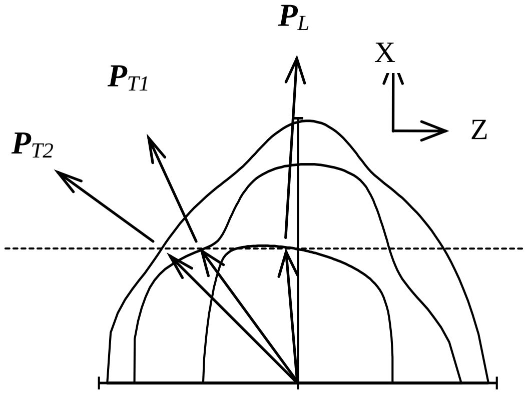

Fig.3. Cross-section of the slowness surface and the Poynting vectors for three bulk modes, which compose supersonic non-leaky wave. X is a propagation direction, Z is an outward normal to the boundary plane.

The analysis of Poynting vectors for three bulk modes, which compose non-leaky wave (fig.3) shows, that two of them, quasi-transverse, take energy from the surface into the bulk, while the third, quasi-longitudinal, brings it to the surface. Therefore, such combination of bulk waves may be found as the special solution of the reflection problem, when the energy of incident quasi-longitudinal bulk wave is completely transformed into the energy of reflected quasi-transverse waves. The corresponding incidence angle is sometimes called "the Brewster's angle"[9], by analogy with optics. In non-piezoelectric medium such special solutions occur in 3D subspace of 4D space (Euler angles, velocity). In piezoelectric crystals the additional condition $|C_I|=0$ must be satisfied and 3D subspace is reduced to 2D one. The cuts, for which CBW satisfies the boundary conditions, may be easily found through the analysis of bulk waves in the infinite crystal, similar to EW lines.

III. Analysis of Composite Bulk Waves in $LiNbO_3$ and Quartz

The CBW propagating at supersonic velocity V in crystal orientation, defined by the Euler angles (φ,θ,ψ), satisfies stress-free mechanical boundary condition, if three vectors

$$\mathbf{T}_i = (\mu_{ij}^{L} m_j , \mu_{ij}^{T1} m_j , \mu_{ij}^{T2} m_j), \quad (i=1,2,3) \qquad (2)$$

are coplanar, where **m** is the unit vector normal to the boundary plane and the stress tensor μ for the specified partial wave having particle displacement **U** and electrostatic potential φ, is defined by

$$\mu_{ij} = c_{ijkl}\, u_k\, n_l + e_{nij}\, n_n\, \varphi \ , \qquad (3)$$

where c_{ijkl} and e_{nij} are respectively, the tensors of elastic stiffness and piezoelectric constants.

The electrical boundary condition is satisfied if CBW has zero electrostatic potential φ (metallized surface) or zero normal component of electrical displacement **D** (free surface). This requirement is equal to the coplanarity between vector

$$\mathbf{\Phi} = (\varphi^{L} ,\ \varphi^{T1} , \varphi^{T2}) \quad \text{(metallized surface)} \ (4)$$

or

$$\mathbf{D}_m= (D_i^{L}\, m_i,\ D_i^{T1}\, m_i,\ D_i^{T2}\, m_i) \ \text{(free surface)} \ (5)$$

and three vectors $\mathbf{T}_i$.

For orientations of $LiNbO_3$, defined by the Euler angles (90°, 90°, ψ), the angular dependence of velocity V(ψ) has been found (fig.4) for CBWs, which satisfy the stress-free boundary condition. This dependence may be considered as a projection of 3D subspace for specified Euler angles φ and θ. Electrical boundary conditions for free and metallized surface are also fulfilled if ψ=163.3°. This angle is in a good agreement with the numerical results obtained in [4].

Though the CBWs, shown in fig.4, satisfy only the mechanical boundary condition, their velocities are very close to that of leaky waves, propagating along the free surface (see fig.4 in [4]). To satisfy the electrical condition it is necessary to admix the non-homogeneous partial wave, which results in non-zero attenuation coefficient. The piezoelectric coupling is zero for non-leaky waves and grows, when $|C_I|$ and δ increase. Therefore, for this type of leaky waves, high piezoelectric coupling may be obtained only if the attenuation is also sufficiently high.

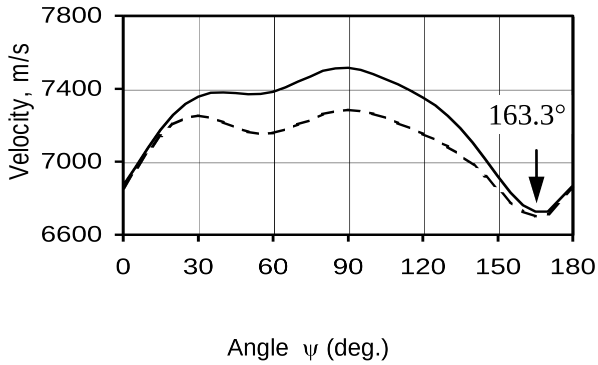

Fig.3. Velocities of composite bulk waves (solid line), propagating in (90°, 90°, ψ) orientations of $LiNbO_3$, and satisfying stress-free boundary condition, versus angle ψ. The point, where electrical boundary conditions are also satisfied, is indicated by the arrow. Dashed line shows the velocities of quasi-longitudinal SSBWs.

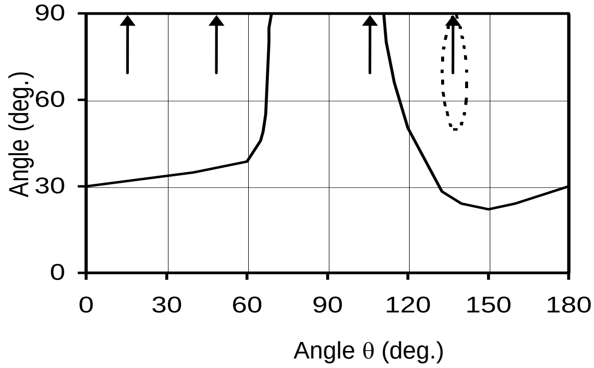

Fig 5. The subspace ψ(θ) (solid line) of cuts in quartz, Euler angles (0, θ, ψ), where supersonic non-leaky wave exists on metallized surface. Inside the area, bounded by the dashed line, leaky waves are transonic ($V_{T2}< V<V_L$). The points, where non-leaky wave becomes one-partial EW, are indicated by arrows.

In quartz the line of orientations θ(ψ) was found for Euler angles (0, θ, ψ), along which the CBWs satisfy both mechanical and electrical (metallized surface) boundary conditions (fig.5). Here leaky waves have zero attenuation coefficient, being non-leaky. For all ψ≠90° non-leaky wave is three-partial.

If ψ=90°, then the non-leaky wave becomes two-partial and comprises incident quasi-longitudinal and only one reflected quasi-transverse bulk modes (L+T), both polarized in the sagittal plane. These solutions exist for any θ. The ratio between the amplitudes of two bulk modes depends on the angle θ (fig.6). In selected four points: θ=15.8°, 47.8°, 105.8° and 137.8° the two-partial solution becomes one-partial, since here the quasi-longitudinal bulk wave is exceptional [5,6].

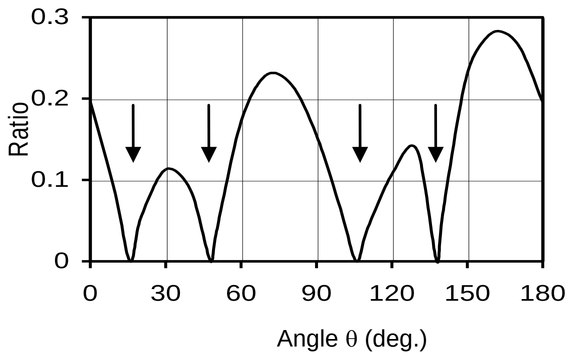

Fig. 6. The amplitude ratio $|C_T|/|C_L|$ between *two partial modes, which compose non-leaky waves, propagating in quartz, Euler angles (0°, θ, 90°), versus angle θ. EW orientations are indicated by arrows.*

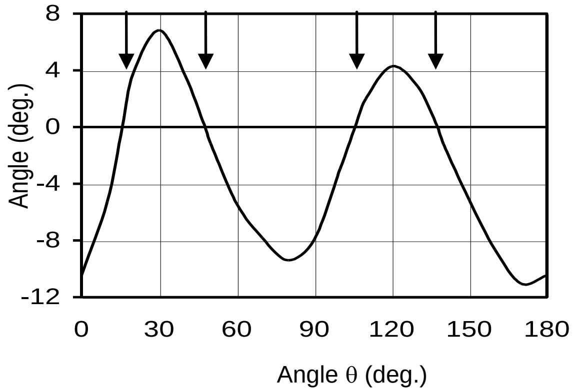

Fig.7. The angle of polarization β for CBW (non-leaky wave), propagating in quartz cuts (0°, θ, 90°), versus angle θ. EW orientations are indicated by arrows.

Fig.7 illustrates the rotation of polarization angle $\beta=\tan^{-1}(U_\perp/U_{||})$ for CBWs, propagating in quartz orientations (0°, θ, 90°). Here $U_\perp$ and $U_{||}$ are respectively normal and tangential components of the displacement vector. Four EWs are pure longitudinal (β=0) and propagate at velocity $V= V_L$. In 4D space EW lines belong to the subspace of non-leaky supersonic waves and correspond to the boundary of this subspace on the side of minimum velocity $V=V_L$.

IV. Conclusions

The method of exceptional waves, earlier developed to search for low-attenuated leaky waves, has been extended to supersonic velocity region. EW is now considered as degenerate composite bulk wave, which is three-partial in general. The subspace, where CBWs satisfy boundary conditions, as well as EW lines, may be found through the analysis of bulk waves in crystal. The method gives all orientations, where supersonic leaky waves can propagate with zero attenuation.

Acknowledgments

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