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Analysis of Leaky Surface Waves in Crystals with Strong Acoustic Anisotropy

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Abstract - The peculiarities of LSAW analysis in crystals with strong acoustic anisotropy are described, when there is a concavity of the slowness surface in the sagittal plane. The additional real roots of characteristic equation appearing in the region of concavity, are involved in LSAW structure and can build new leaky solution. The analysis of LSAWs has been performed in some orientations of quartz in terms of contributing partial bulk waves. The continuous transformation of the slowness surface following from the rotation of the sagittal plane, changes gradually the nature of LSAW from the surface-guided mode to the solution of the reflection problem for bulk waves. The concavity of the slowness surface is shown to cause the splitting of LSAW branch. The composite exceptional wave, earlier known to exist in symmetric orientations of some crystals, has been found in non-symmetric orientation.

I. INTRODUCTION

The previous study of leaky surface acoustic waves (LSAWs) based on the analysis of acoustic anisotropy [1-4], has revealed the close connection between the properties of LSAW propagating in the specified orientation and the behavior of bulk acoustic waves in the corresponding sagittal plane. A preliminary numerical investigation of acoustic anisotropy can be very helpful for further LSAW analysis(see, for example, [1]). Such investigation includes the calculation of slownesses (inverse phase velocities) of the bulk waves as functions of propagation direction in the sagittal plane. Then the limiting velocities are determined for each bulk wave branch. Any limiting velocity corresponds to the bulk wave carrying energy along the boundary surface. If such a wave, while propagating along the surface, leaves it stress-free, it is called “exceptional wave” (EW) and can give a quasi-bulk leaky wave with small or zero attenuation.

The number of limiting velocities depends on the geometry of slowness section in the sagittal plane called “transonic state”. Six types of transonic states were established in [5] and the possibility of secluded “true” SAW solutions in leaky wave velocity region was later examined for different types. Both EW and “true” SAW propagate faster than common SAW and hence they are very promising for high-frequency applications.

One more important aspect of LSAW analysis was considered in [4]. It is the connection between the solutions of reflection problem for bulk waves and the behavior of leaky waves in crystals. Earlier the relation between the theory of reflection and the SAW theory was extensively studied (for example [6-7]) and some important theorems were proved.

The present paper is a further study of the behavior of LSAWs in crystals. A material with strong acoustic anisotropy is analyzed, in which the negative curvature (concavity) of the slowness surface occurs for some of bulk wave branches and hence the additional incoming and outgoing bulk waves can be involved in LSAW solutions. The behavior of leaky waves is analyzed in terms of contributing bulk waves.

II. THEORETICAL BACKGROUND

For applications in SAW devices, leaky waves with low attenuation are most attractive. The analysis of numerical data for leaky waves in different crystals has been performed and some published theoretical results have been generalized in order to classify non-attenuated leaky solutions found in different crystals. Thus three possibilities for leaky wave to become non-leaky have been identified.

First one is a vanishing of amplitude coefficient for bulk partial wave (or waves) involved in LSAW solution. The remaining three-partial or two-partial inhomogeneous wave does not attenuate while propagating along the surface since its structure is similar to that of Rayleigh surface wave. Such a solution may be called “true” SAW and its energy is usually localized within a narrow layer near the surface. As it was proved in [6,7], the true SAW solutions can be found in some isolated points of 4D space (Euler angles, velocity), when the propagation velocity is within the interval $V_{L1}<V<V_{L2}$ and require certain relations between material constants for their existence if $V_{L2}<V<V_{L3}$. Here V_{L1},V_{L2} and V_{L3} are the limiting velocities of slow quasishear, fast quasishear and quasilongitudinal bulk waves in the specified sagittal plane, and usually $V_{L1}<V_{L2}<V_{L3}$. If $V>V_{L3}$, the inhomogeneous SAW solution would be one-partial and it can not satisfy the boundary conditions[6]. Therefore, the probability to find “true” SAW solution decreases with increasing propagation velocity. However, such solutions can be found in some crystals, not only in symmetric cuts but also in non-symmetric ones. For example, such waves can propagate in quartz orientations with Euler angles (90°, 90°, 117°) [8] and (0°, 15°, 0°). The last orientation is close to LST-cut.

The second possibility for LSAW to become non-attenuated is degeneration into EW. This type of non-leaky solutions also can be found both in non-symmetric and symmetric cuts, and the permitted orientations form the lines in 4D space of non-piezoelectric crystal [2,3]. In contrast to true SAW, it is a bulk wave with homogeneous distribution of energy along the depth of crystal. However, if any kind of perturbation (piezoelectric effect, mass-loading effect, misorientation etc.) shifts the propagation velocity from the limiting to lower values, the prevailing homogeneous partial wave becomes slightly inhomogeneous and three other partial modes are admixed. If the "generating" EW is slow quasishear with velocity V_{L1}, the perturbed solution becomes pure inhomogeneous with the energy concentrated near the surface. Such a solution can be efficiently generated and detected on the surface. If the "generating" EW is fast quasishear or quasilongitudinal, in addition to inhomogeneous partial waves, the perturbed solution comprises bulk waves with lower propagation velocities and can become non-leaky only if the bulk partial waves are uncoupled. The examples of perturbed EWs are widely used LSAWs in 41°-YX $LiNbO_3$,36°-YX $LiTaO_3$ and quasilongitudinal LSAW in $Li_2B_4O_7$ orientation with Euler angles (45°,46°,90°) [1].

And the third possibility for LSAW solution to degenerate into non-leaky wave was considered in [4]. It is valid for high-velocity waves because requires at least two homogeneous bulk partial waves including incoming to the surface (incident) and outgoing from the surface into the bulk (reflected) modes. Though the Poynting vectors corresponding to these bulk modes are inclined with respect to the boundary plane, the resulting composite bulk wave (CBW) propagates along the surface and can satisfy the boundary conditions. This type of non-leaky waves can be found as a special solution of the reflection problem provided that the amplitudes of all inhomogeneous partial waves tend to zero and the number of partial waves, involved in the reflection problem, is reduced from 5 to 4 or less.

For these waves, the area of existence depends on the propagation velocity. If $V>V_{L3}$ and the crystal surface is metallized, it is 2D subspace of 4D space. Hence such supersonic solutions can be easily found in many crystals [4]. If $V_{L2}<V<V_{L3}$, only two bulk waves can build CBW solution and the area of existence is reduced. The existence condition for CBW on metallized surface was considered in [4]. When the surface is free (non-metallized), the existence condition for pure CBW is more strict because of additional partial wave of electric potential in vacuum. Therefore, pure CBW solutions can be found on metallized surface rather than on free one.

The main feature which distinguishes CBW from the other types of non-leaky waves is substantially bulk nature. Indeed, no perturbation can appreciably change this nature unless it is followed by the considerable growth of attenuation coefficient. Therefore, this type of waves is not very attractive for SAW devices. On the other hand, the continuous rotation of the crystal plane or the propagation direction can cause the transformation of CBW into EW, as it occurs in quartz [4] and in spite of their bulk nature, such CBWs can be implemented in SAW devices. For example, low-attenuated LSAW which propagates in ST,X+25° cut of quartz and was proved to be a perturbed CBW [4], was generated and detected by interdigital transducers in [9].

It is important to point out that in general neither CBW nor EW can be referred to surface waves. Nevertheless, in the vicinity of selected orientations the attenuation coefficient grows and leads to localization of the wave energy near the surface. Moreover, further rotation of the crystal plane or the propagation direction can cause the transformation of one non-leaky type into another, surface-guided. Therefore, herein all these waves are called LSAWs.

There is one more reason for including CBWs in the general analysis of leaky waves in crystals. The behavior of non-leaky solution, first of all the relation between its piezoelectric coupling and attenuation coefficient, depends on the wave nature and can be predicted after the analysis of the wave structure and its identification.

It must be noted that the present paper is aimed only at the demonstration of different types of non-leaky waves existing in real crystals and is based on the numerical techniques. The problem of rigorous theoretical analysis of existence conditions for leaky waves with zero and small attenuation is a subject for future study. The estimation of piezoelectric coupling for low-attenuated leaky waves based on the analysis of their structure, would be especially important in the search for useful orientations for SAW devices.

III. Analysis of LSAWs in Quartz, Euler Angles (0°, 50°, ψ)

The search for degeneration points (acoustic axes) on the slowness surface is an important step in the analysis of acoustic anisotropy of any crystal. If the found propagation direction for two bulk waves with equal phase velocities is not parallel with 4-fold or 6-fold symmetry axis, one of two degenerate branches usually exhibits concavity around the degeneration point. If this concavity occurs in the sagittal plane, the solution of characteristic equation gives the additional pair of real roots. Each pair corresponds to one incoming and one outgoing waves. It should be noted that the terms "incoming" and "outgoing" refer to the directions of Poynting vectors and not to the wave vectors as some authors do. Obviously, these additional bulk waves must be involved in LSAW solutions. Thus, the situation can be realized when the wave with the structure similar to that of quasilongitudinal wave travels at the velocity typical for quasishear waves.

In order to study the effect of slowness surface concavities on the behavior of LSAWs, the quartz orientations with Euler angles (0°, 50°, ψ) have been analyzed. Quartz is known as a crystal with strong acoustic anisotropy. In addition to degeneration in the symmetry plane YZ, which is typical for any crystal of the same symmetry class, there exists one more degeneration point of general position on its slowness surface [3]. Both degeneration points are close to the orientations analyzed here.

The velocities and attenuation coefficients of leaky waves on metallized surface are depicted in Fig.1 as functions of angle ψ. The dashed lines show the limiting velocities resulting from the analysis of slowness surface. The slow quasishear branch exhibits the concavity within the interval $\psi\approx$20-70° giving two additional limiting velocities. The concavity region for fast quasishear wave has been found for $\psi\approx$0-28°. Besides, within the interval $\psi\approx$25-36° the limiting velocity of additional slow quasishear wave occurs to be larger than that of the fast quasishear wave. Consequently, three bulk partial waves can contribute LSAW solution instead of one.

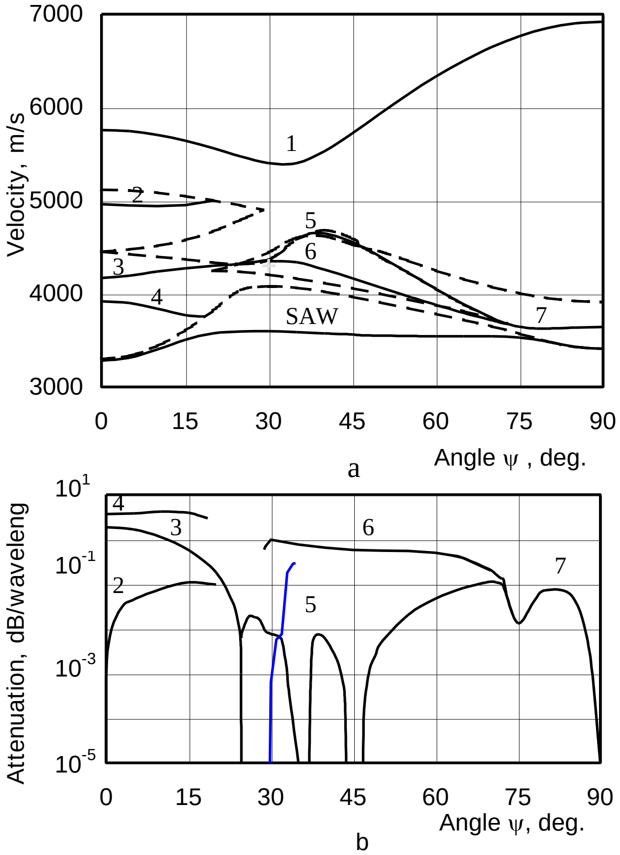

Fig.1. V*elocities (a) and attenuation coefficients (b) of SAW, leaky solutions(solid lines) and limiting bulk waves (dashed lines) in quartz cuts (0°, 50°, ψ) versus angle ψ.*

The multi-variant search for leaky solutions has been performed, that is when the number of contributing bulk partial waves is equal or more than two, all possible combinations of incoming-outgoing waves have been considered. The only restrictions imposed on leaky solution

$$\mathbf{U}=\sum_{n=1}^{4} C_n \mathbf{U}_{on} \exp[j(kx-\omega t)] \exp(-\delta kx) \exp(j\alpha_n z) \quad (1)$$

are the positive attenuation coefficient δ and at least one partial wave with negative imaginary part of α, providing the leak of energy into the bulk. In (1), the axes X and Z are parallel with the propagation direction and with the inner normal to the boundary plane, respectively.

The leaky waves found in this way would obviously include 4-partial solutions of the reflection problem. On the other hand, this approach provides the continuity of each LSAW branch and reveals the gradual transformation of the wave nature caused by the rotation of the sagittal plane. The problem of physical acceptability of found solutions is further considered.

All found LSAW branches are numbered in Fig.1. The line 1 refers to quasilongitudinal leaky waves. The attenuation coefficients are not presented for these waves because they were earlier examined in [4]. It should be mentioned only that the velocities of these waves are supersonic $V>V_{L3}$ for any ψ.

The lines 3,4 and 7 refer to the well known type of quasishear LSAWs with the single outgoing homogeneous partial wave. The waves 3 and 4 have similar structure though their velocities and attenuation coefficients are different. One of these waves (line 4) is modified into the strongly attenuated surface skimming bulk wave when ψ approaches 18.5°. The attenuation coefficient of the other wave (line 3) tends to zero when ψ approaches 25.14° and further this branch is splitted into several LSAWs. For example, in the point ψ=30° three leaky solutions have been found with very

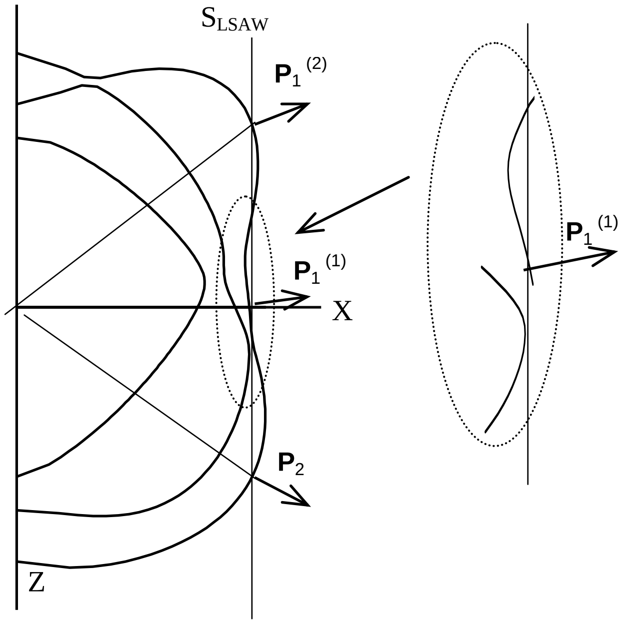

Fig.2. Cross-section of the slowness surface by the sagittal plane in quartz cut with Euler angles (0°, 50°, 30°). Poynting vectors $\mathbf{P_1}$ *(two variants) and* $\mathbf{P_2}$ *refer to two contributing partial bulk waves when the propagation velocity of leaky wave is* V_{LSAW}.

close velocities and different attenuation coefficients. One of these solutions is nearly non-attenuated. Its structure is illustrated in Fig.2, where the slowness section is depicted in the sagittal plane. The vertical line $S_{LSAW}=1/V_{LSAW}$ intersects the slowness surface in four points giving two outgoing and two incoming bulk waves, all slow quasishear. But only two of arising bulk waves can contribute LSAW solution. For the analyzed non-leaky wave, they are indicated by the Poynting vectors $\mathbf{P}_1^{(1)}$ and $\mathbf{P}_2$. The first wave is incoming while the second one is outgoing. Changing the propagation angle ψ slightly, one can expect the growth of attenuation. Consequently, all partial waves become inhomogeneous. Now the incoming partial wave has the amplitude which decreases with depth (Im(α)>0) and the amplitude of the outgoing wave becomes growing with depth (Im(α)<0), which is consistent with the physical nature of both waves. The other LSAW solution can be built at the same or close slowness value, with Poynting vector $\mathbf{P}_1^{(2)}$, also corresponding to incoming wave, instead of $\mathbf{P}_1^{(1)}$. This solution gives LSAW branch 6.

The largest number of leaky solutions can be constructed within the interval ψ≈25-36°, where three bulk waves can be involved in low-velocity solution. The continuous transformation of the slowness section with increasing ψ is depicted in Fig.3. For ψ<25° (Fig. 3a), the vertical line $S=S_{LSAW}$ gives no intersections with both quasi-shear slowness branches within the shown area and the only contributing bulk mode with the Poynting vector $\mathbf{P}_2$ (Fig.2) is responsible for the leak of energy into the bulk. For ψ=25.14°, two limiting velocities become equal and LSAW solution can be constructed with two bulk partial waves carrying energy along the surface (Fig.3b). Such solution can be non-attenuated under certain conditions. Finally, for ψ=25.14-36.5°, in addition to

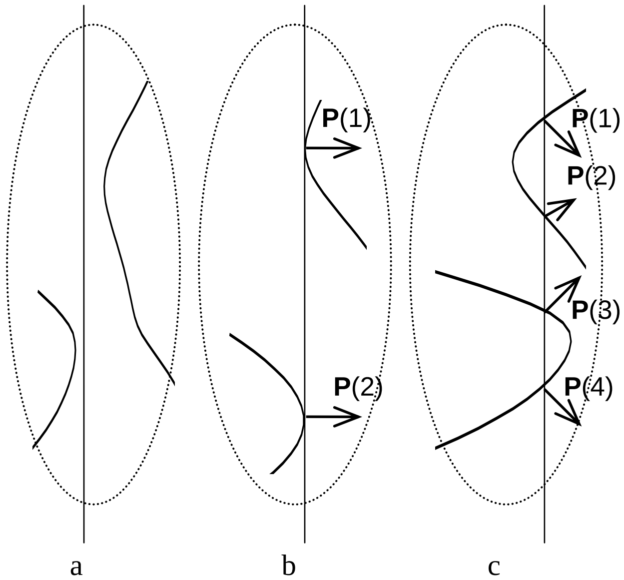

Fig.3. Fragments of slowness sections when there is a concavity for slow quasishear branch. Leaky solution includes (a)none of two bulk waves, (b)two limiting waves, (c)incoming and outgoing bulk waves.

the outgoing wave with Poynting vector $\mathbf{P}_2$ (fig.2), LSAW involves one more outgoing wave and one incoming wave, both must be chosen from 4 possible bulk modes (Fig.3c). The found solution involves the bulk waves with the Poynting vectors **P**(1) and **P**(3).

The slowness sections for orientations ψ=36.5° and ψ=47° include the points which are close to degeneration of quasishear waves. Degenerate bulk wave can have any Poynting vector $\mathbf{P}_1$ belonging to the cone in Fig.4, dependent on its polarization. As it was shown in [3], three of these degenerate bulk waves are exceptional. According to [4], EW point belongs to the space line of CBWs. Therefore, non-attenuated solutions of CBW type have been found when ψ approaches to 36.5 and 47°. As a result of slight misorientation from degeneration point, the bulk mode with the Poynting vector $\mathbf{P}_2$ is admixed to EW.

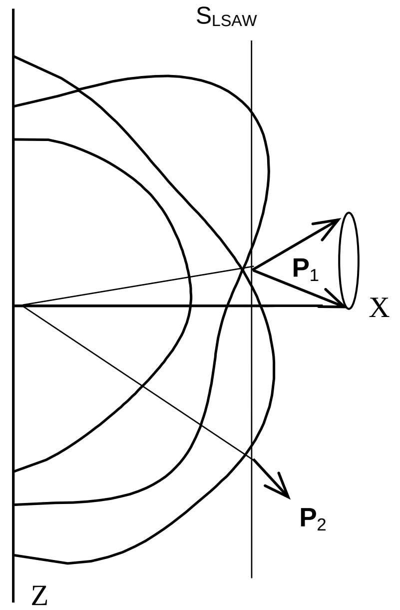

Fig.4. Slowness section in quartz, Euler angles (0°, 50°, 47°). The cone of Poynting vectors $\mathbf{P}_1$ *corresponds to degenerate quasishear bulk waves.*

A special solution of boundary problem can be built with two bulk modes belonging to the same slowness branch, when the sagittal plane is normal to the plane of crystal symmetry or parallel with the even-fold symmetry axis. In quartz it is true for orientations with Euler angles (0°, θ ,0°) and (90°, 90°, ψ) (Fig.5). The two-partial non-attenuated solution, including one outgoing and one incoming bulk modes, which are symmetric with respect to the boundary plane, is called the "simple reflection" [10]. In orientations (0°, θ ,0°), the normal stresses for two bulk modes T_{33} cancel due to symmetry and the "simple reflection" solution requires only $T_{31}= T_{32}=0$, while in orientations (90°,90°,ψ), $T_{31}=T_{32}=0$, and the existence condition for "simple reflection" is $T_{33}=0$. Such solutions have been found in some crystals [11].

The "simple reflection" solutions exist in quartz cuts (0°,θ,0°) including θ=50°. While ψ grows, it gives one more LSAW branch (line 2) in the concavity region. These waves are strongly attenuated and of no practical importance. However, in the concavity region under certain conditions the "simple reflection" solution can degenerate into the so-called "composite exceptional wave"(CEW) [11] (see Fig.5, fragment). If the velocity of "simple reflection" solution tends to the limiting value, both Poynting vectors of the reflected and the incident bulk waves become parallel with the surface. Even if none of these bulk waves is exceptional, their combination can satisfy the stress-free boundary condition. The behavior of CEW is similar to that of simple EW, and the perturbation also can make CEW inhomogeneous wave, which is efficiently generated on the surface.

CEWs can propagate in some crystals, for example, in $LiTaO_3$, orientation (90°,90°,148°) and $Li_2B_4O_7$, orientation (45°,90°,70°). Though such waves do not exist in quartz, it can be shown that for any CEW propagating in rotated Y-cut along X-axis in crystal with quartz-like symmetry (class *32*), shear displacements cancel and such CEW must be pure longitudinal though the contributing bulk waves are quasishear.

Another type of CEW, non-symmetric one, to the best author's knowledge, has not been reported earlier. It is constructed from two limiting bulk waves belonging to different slowness branches as it is shown in Fig.3b. In the quartz orientation (0°, 50°, 25.14°) two bulk waves with the limiting velocities are prevailing, while in orientation (0°, 49°, 26.45°) the amplitudes of two other partial waves tend to zero and the corresponding solution is a non-symmetric two-partial CEW.

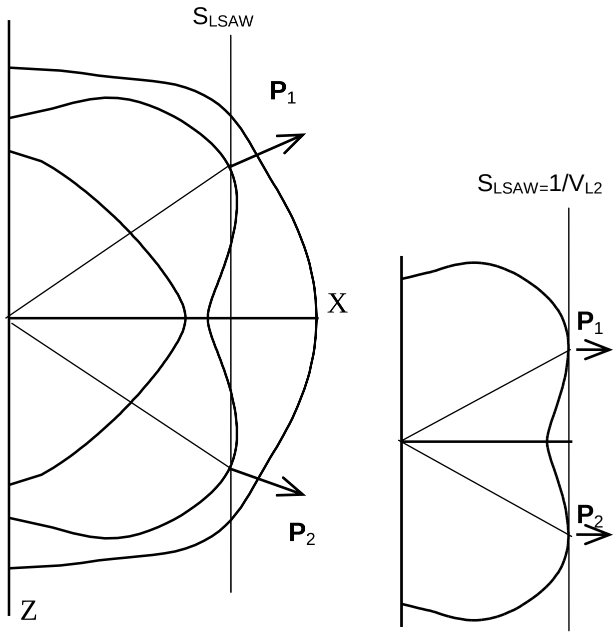

Fig.5. Slowness section in quartz, Euler angles (0°, 50°, 0°), with simple reflection solution for fast quasishear mode. In the fragment this solution degenerates into CEW.

The displacement components of LSAW have been calculated as functions of depth for two orientations, (0°, 50°, 25.14°) and (0°, 50°, 25.0°) (Fig.6a,b), when the surface is metallized. The small misorientation is shown to reduce the penetration depth, and in the second cut LSAW is a well-behaved surface-guided mode. To verify the physical acceptability of found LSAWs, the components of instantaneous power flow **P** have been calculated as functions of depth. The results are presented in Fig.7a,b. In both analyzed cuts, P_3 >0 which is consistent with the conservation of energy requirement [12]. Thus these solutions are physically acceptable. In orientation (0°, 50°, 25.14°), P_3 is close to zero because of nearly zero attenuation.

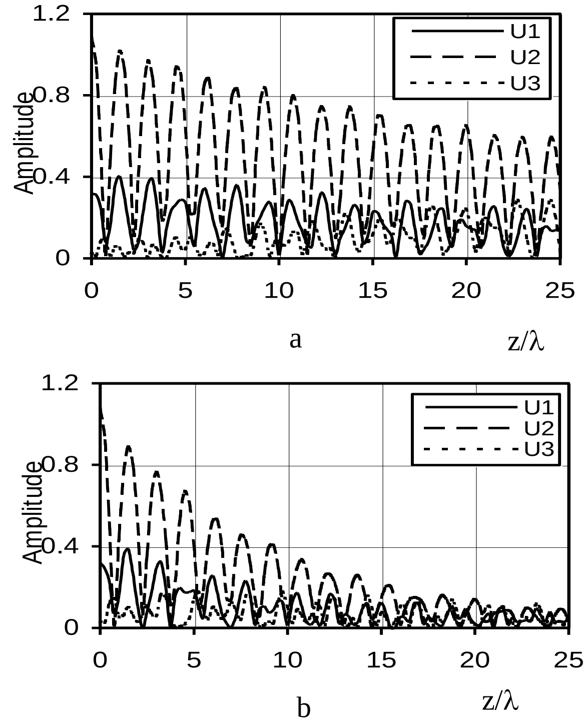

Fig.6. Displacement components versus depth for LSAWs propagating in quartz, Euler angles (0°, 50°, 25.14°) (a) and (0°, 50°, 25.0°) (b).

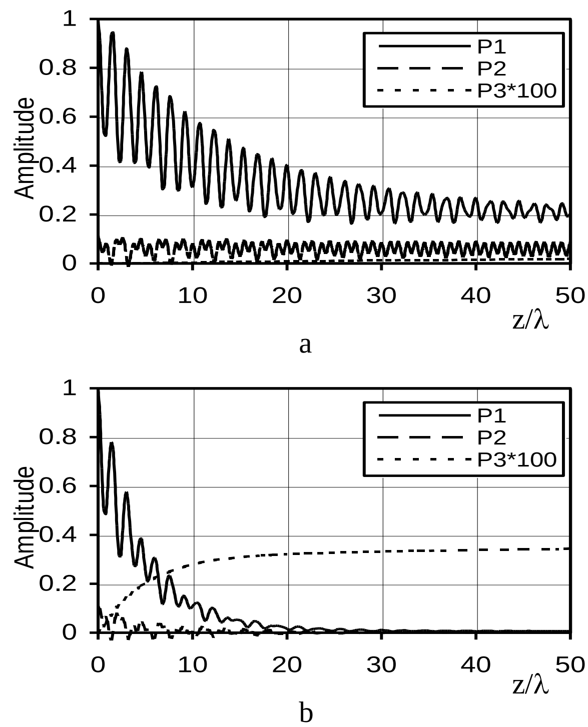

Fig.7. Poynting vector components versus depth for LSAWs propagating in quartz, Euler angles (0°, 50°, 25.14°) (a) and (0°, 50°, 25.0°) (b).

However, P_1 and P_2 do not tend to zero in the depth of crystal, which proves the bulk nature of LSAW. On the contrary, in orientation (0°, 50°, 25.0°) P_3 grows because of energy leak into the bulk, while P_1 and P_2 vanish at the depth about 50λ, where λ is LSAW wavelength.

The variation of instantaneous power flow direction versus depth can be estimated using azimuth ϕ and declination ξ angles [13](Fig.8a,b). In orientation (0°,50°, 25.0°), LSAW is gradually transformed into the outgoing bulk wave when z>100λ and both angles correspond to this bulk wave. In orientation (0°, 50°, 25.14°), $\xi \approx 0$ (no leak of energy), but LSAW is transformed into the combination of incoming and outgoing bulk waves and $\phi(z)$ exhibits the interference behavior at any depth.

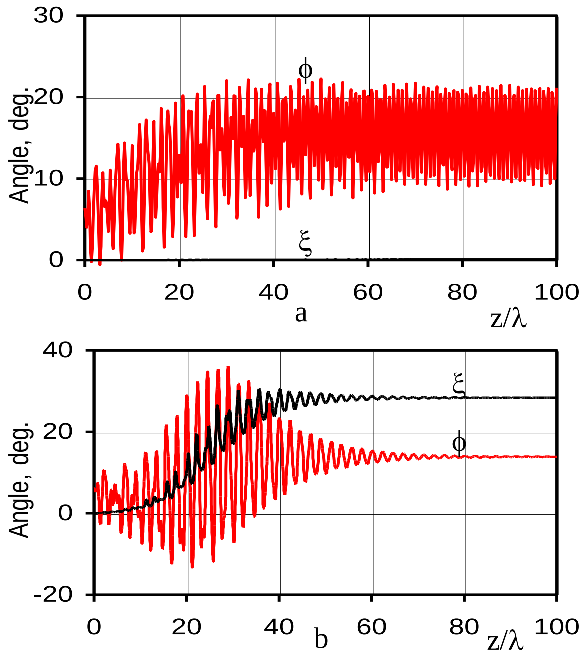

Fig.8. Azimuth ϕ and declination ξ angles of Poynting vector versus depth for LSAWs propagating in quartz, Euler angles (0°, 50°, 25.14°) (a) and (0°, 50°, 25.0°) (b).

The depth behavior of P_3 shown in Fig.7b is typical for any LSAW from Fig.1. Therefore, none of these solutions can be considered non-physical, though only some of them are surface-guided waves with low attenuation coefficient and small penetration depth. All found LSAWs comprise either one outgoing bulk wave or both outgoing and incoming bulk waves. This is consistent with theoretical results obtained in [6].

IV. Conclusions

An efficient numerical approach to the analysis of leaky waves in crystals with strong acoustic anisotropy has been developed. It is especially helpful when there is a concavity of slowness surface in the sagittal plane and reveals the continuous transformation of surface-guided leaky waves into the combination of bulk waves. The multi-variant numerical procedure proves the intimate relation between LSAW problem and the reflection problem for bulk waves.

Acknowledgments

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